1.1 (40 pts.) Let $x(t) = (2-j)\sin(11\pi t) - (j+3)\cos(2\pi t)$ and $y(t) = \sin(11\pi t) + (5-j)\cos(2\pi t)$.

(a) Find the period T_o and fundamental frequency f_o for $x(t)$ and $y(t)$.

(b) Expand $x(t)$ and $y(t)$ in complex Fourier series valid for all t.

(c) Find the power in $x(\cdot)$ and the power in $y(\cdot)$.

(d) Evaluate

$$\langle x, y\rangle = \frac{1}{T_o}\int_0^{T_o} x(t)y^*(t)\,dt.$$

1.2 (20 pts.) Let $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f_o t}$, and let $g(t) = j2x(t-2)\sin(10\pi f_o t)$. Find the complex Fourier series for $g(t)$; i.e., find G_n in terms of the X_ns.

1.3 (20 pts.) An LTI system has frequency response $H(f) = 2\text{rect}(\frac{f+20}{2}) + 2\text{rect}(\frac{f-20}{2})$. Find the impulse response $h(t)$.

1.4 (20 pts.) Evaluate

$$y(t) = \int_{-\infty}^{\infty} 2\text{sinc}(t-\tau)\text{sinc}(3\tau)\,d\tau.$$

$\boxed{1.1a}$ Jonathan Kotta

period of $\sin(11\pi t) = \frac{11}{2}$

period of $\cos(2\pi t) = 1$

$\frac{35}{40}$

$T_0 = m \frac{2}{11} = n \cdot 1$, $m, n \in \mathbb{Z}$ $m = 11, n = 2$

$\frac{93}{100}$

$T_0 = 2$ ✓

$f_0 = \frac{1}{2}$ ✓

x and y have the same period

$\boxed{1.1.b}$

$$x = (2-j)\sin(11\pi t) - (3+j)\cos(2\pi t)$$

$$= \left(\frac{2-j}{2j}\right)\left(e^{2\pi j t\, 11/2} - e^{-2\pi j t\, 11/2}\right) - \frac{(3+j)}{2}\left(e^{2\pi j t\, 2/2} + e^{-2\pi j t\, 2/2}\right)$$

$$X[n] = \begin{cases} -\frac{1}{2} - j ✓, & n = 11 \\ -\frac{3}{2} - \frac{1}{2}j ✓, & n = 2 \\ -\frac{3}{2} - \frac{1}{2}j ✓, & n = -2 \\ +\frac{1}{2} + j ✓, & n = -11 \\ 0 ✓, & \text{else} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X[n] e^{2\pi j t n/2}$$ ✓

$$y = \sin(11\pi t) + (5-j)\cos(2\pi t)$$

$$= \frac{1}{2j}\left(e^{2\pi j t\, 11/2} - e^{-2\pi j t\, 11/2}\right) + \left(\frac{5-j}{2}\right)\left(e^{2\pi j t\, 2/2} + e^{-2\pi j t\, 2/2}\right)$$

$$Y[n] = \begin{cases} -\frac{1}{2}j ✓, & n = 11 \\ \frac{5}{2} - \frac{1}{2}j ✓, & n = 2 \text{ or } n = -2 \\ +\frac{1}{2}j ✓, & n = -11 \\ 0 ✓ & \text{else} \end{cases}$$

$$y(t) = \sum_{n=-\infty}^{\infty} Y[n] e^{2\pi j t n/2}$$

$\boxed{1.1.c}$ Jonathan Kotte

$$P_x = \sum_{n=-\infty}^{\infty} |X[n]|^2 = \left(\left(\frac{1}{4}+1\right) + \left(\frac{9}{4} + \frac{1}{4}\right)\right)2 = \frac{9+4+2}{2} = \boxed{\frac{15}{2}}$$

$$P_y = \sum_{n=-\infty}^{\infty} |Y[n]|^2 = \left(\frac{1}{4} + \left(\frac{25}{4} + \frac{1}{4}\right)\right)2 = \boxed{\frac{27}{2}}$$

$\boxed{1.1.d}$

$$\frac{1}{T_0}\int_0^{T_0} x(t)\,\overline{y(t)}\,dt = \sum_{n=-\infty}^{\infty} X[n]\,\overline{Y[n]}$$

$+\frac{1}{2}$ X $\qquad -\frac{1}{4}j + \frac{1}{2}$

$$= \left(\cancel{-}+\frac{1}{4}j - \frac{1}{4}\right) + \left(\left(-\frac{15}{4} - \frac{1}{4}\right) + j\left(-\frac{5}{4} + \frac{3}{4}\right)\right)\cdot 2 + \left(\frac{1}{4}j - \frac{1}{4}\right)$$

X

−5

$$= -\frac{1}{2} + \frac{1}{2}j + -\frac{16}{2} + j\frac{-2}{2} = \boxed{-\frac{17}{2} + -\frac{1}{2}j}$$

X

$\boxed{1.2.\text{[illegible]}}$

$$g(t) = j2x(t-2)\sin(10\pi f_0 t) = x(t-2)\left(e^{2\pi j 5 f_0 t} \cancel{+} - e^{-2\pi j 5 f_0 t}\right)$$

20/20

$$\mathcal{F}\{x(t)\} = X[n]$$

$$\mathcal{F}\{x(t-2)\} = e^{-2\pi j n 2 f_0} X[n]$$

$$\mathcal{F}\{g(t)\} = \boxed{X[n-5]\,e^{-2\pi j(n-5)2f_0} - X[n+5]\,e^{-2\pi j(n+5)2f_0} = G[n]}$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} G[n]\,e^{2\pi j n t f_0}}$$

Jonathan Ketta

$\boxed{1.3}$

impluse response = $h(t) = \mathcal{F}^{-1}\{H(f)\}$

18/20

$$\mathcal{F}^{-1}\{\text{rect}(f)\} = \text{sinc}(t)$$

$$\mathcal{F}^{-1}\{\text{rect}(\tfrac{f}{2})\} = 2\,\text{sinc}(2t)$$

$$\mathcal{F}^{-1}\{\text{rect}(\tfrac{f+20}{2})\} = 2\,\text{sinc}(2t)e^{-2\pi j 20t}$$

$$\mathcal{F}^{-1}\{H(f)\} = 4\,\text{sinc}(2t)e^{-2\pi j 20t} + 4\,\text{sinc}(2t)e^{+2\pi j 20t}$$

$$= \boxed{\cancel{2}\,8\,\text{sinc}(2t)\cos(2\pi 20t) = h(t)}$$

$\boxed{1.4}$

$$y = \int_{-\infty}^{\infty} 2\,\text{sinc}(t-\tau)\,\text{sinc}(3\tau)\,d\tau = 2\,\text{sinc}(t) * \text{sinc}(3t)$$

20/20

$$= \mathcal{F}^{-1}\{\mathcal{F}\{2\,\text{sinc}(t)\}\cdot\mathcal{F}\{\text{sinc}(3t)\}\}$$

$$\mathcal{F}\{2\,\text{sinc}(t)\} = 2\,\text{rect}(f)$$

$$\mathcal{F}\{\text{sinc}(3t)\} = \tfrac{1}{3}\text{rect}(\tfrac{f}{3})$$

$$2\,\text{rect}(f)\,\tfrac{1}{3}\text{rect}(\tfrac{f}{3}) = \tfrac{2}{3}\text{rect}(f)$$

$$\mathcal{F}^{-1}\{\tfrac{2}{3}\text{rect}(f)\} = \boxed{\tfrac{2}{3}\,\text{sinc}(t)} = y(t)$$